DIFFERENTIAL EQUATION OF THE FIRST ORDER BUT NOT OF THE FIRST DEGREE

Ex. 1. $p^2 - p(e^x + e^{-x}) + 1 = 0$

$\Rightarrow p^2 - pe^x - pe^{-x} + 1 = 0$

$\Rightarrow p(p - e^x) - e^{-x}(p - e^x) = 0$

$\Rightarrow (p - e^x)(p - e^{-x}) = 0$

$\Rightarrow p - e^x = 0 \;;\; p - e^{-x} = 0$

$\Rightarrow \cancel{p = e^x,\ p = e^{-x}}$

$\because \; p - e^x = 0$ and $p - e^{-x} = 0$

$\Rightarrow \frac{dy}{dx} - e^x = 0$

$\Rightarrow \frac{dy}{dx} = e^x \Rightarrow dy = e^x dx$

$\Rightarrow \int dy = \int e^x dx$

$\Rightarrow y = e^x + c$

$\Rightarrow y - e^x - c = 0$

$\Rightarrow \frac{dy}{dx} - e^{-x} = 0$

$\Rightarrow dy - e^{-x} dx = 0 \quad \Rightarrow \int dy - \int e^{-x} dx = 0$

$\Rightarrow y - \left(\frac{e^{-x}}{-1}\right) - c = 0$

$\Rightarrow y + e^{-x} - c = 0$

Hence, the solution is

$(y - e^x - c)(y + e^{-x} - c) = 0$ Ans

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Ex. 2. $p^2 + 2xp - 3x^2 = 0$

$\Rightarrow p^2 - xp + 3xp - 3x^2 = 0$

$\Rightarrow p(p-x) + 3x(p-x) = 0$

$\Rightarrow (p+3x)(p-x) = 0$

$\Rightarrow p+3x = 0 \quad , \quad p-x = 0$

$\Rightarrow \frac{dy}{dx} + 3x = 0 \quad , \quad \frac{dy}{dx} - x = 0$

$\Rightarrow dy + 3x\,dx = 0 \quad , \quad dy - x\,dx = 0$

$\Rightarrow y + \frac{3x^2}{2} - c = 0 \quad , \quad y - \frac{x^2}{2} - c = 0$

Hence, the solution is

$\left(y + \frac{3x^2}{2} - c\right)\left(y - \frac{x^2}{2} - c\right) = 0.$

$\Rightarrow (2y + 3x^2 - c)(2y - x^2 - c) = 0$ Ans

3. $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$

Sol^n $p^2x + p(y-x) - y = 0$

$\Rightarrow p^2x + py - px - y = 0$

$\cancel{\Rightarrow p^2x - p\cancel{x} + py - y = 0}$ $\Rightarrow p^2x - px + py - y = 0$

$\Rightarrow px(p-1) + y(p-1) = 0$

$\Rightarrow (p-1)(px+y) = 0$

$\Rightarrow p - 1 = 0$; $px + y = 0$

$\Rightarrow \frac{dy}{dx} - 1 = 0$, $x\frac{dy}{dx} + y = 0$

$\Rightarrow dy - dx = 0$, $\cancel{xdy + ydx}$ $\Rightarrow x\,dy = -y\,dx$

$\Rightarrow y - x - c = 0$, , $\frac{dy}{y} + \frac{dx}{x} = 0$ $\Rightarrow \log y + \log x = \log c$

$\Rightarrow \log xy = \log c$

$\Rightarrow xy = c \Rightarrow xy - c = 0$

Hence, the solution is

$(y-x-c)(xy-c) = 0$ Ans

Q. $p(p+x) = y(x+y)$

$\Rightarrow p^2 + px - xy - y^2 = 0$

$\Rightarrow (p^2 - y^2) + (px - xy) = 0$

$\Rightarrow (p-y)(p+y) + x(p-y) = 0$

$\Rightarrow (p-y)\{p+y+x\} = 0$

$\Rightarrow p - y = 0$, $\frac{dy}{dx} + y + x = 0$

$\Rightarrow \frac{dy}{dx} - y = 0$

$\Rightarrow \frac{dy}{dx} + y = -x$

$\Rightarrow \frac{dy}{y} = dx$

Here, $P = 1$, $Q = -x$

$IF = e^{\int dx} = e^x$

$\Rightarrow \log y = x + c$

$\Rightarrow \log y - x - c = 0$

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Hence, the sol^n is

$y.e^x = \int -x.e^x \, dx$

$\Rightarrow ye^x = -[x\int e^x dx - \int(\int e^x dx).d(x)]$

$\Rightarrow ye^x = -[xe^x - \int e^x dx] = -[xe^x - e^x] = -e^x(x-1)+c$

$\Rightarrow y = -x + 1 + ce^{-x}$

$\Rightarrow y+x-1 \mp ce^{-x} = 0$ Ans

Hence, the solution is

$(\log y - x - c)(y+x-1\mp ce^{-x}) = 0$ Ans//

4. $\frac{dy}{dx} + \frac{dx}{dy} = 3\frac{1}{3}$ [P.U. 59A]

5. $p^2 - 2p\cosh x + 1 = 0$ [P.U. 63A]

6. $p^2 + p(2x - y) - 2xy = 0$ [Mithila 78A]

7. $p^2 + 2xp - y^2p - 2xy^2 = 0$

8. $p(p^2 + xy) = p^2(x + y)$ [P.U. 63S; R.U. 64S]

9. $p^2x^2 + 3xyp + 2y^2 = 0$

10. $x + yp^2 = p(1 + xy)$

11. $yp^2 + (x - y)p - x = 0$

12. $xyp^2 - (x^2 + y^2)p + xy = 0$

13. $xyp^2 - (x^2 - y^2)p - xy = 0$ [P.U. 64A, 66A; Mithila 76A]

14. $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$ [Bhag. 94H; R.U. 94H]

[**Hint**: L.H.S. = $(px - 2y)(py + 3x)$]

15. $\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = x^2 + xy$ [P.U. 55S]

16. $x^2\left(\frac{dy}{dx}\right)^2 - xy\frac{dy}{dx} = y^2$ [P.U. 50A]

17. $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$

[**Hints**: Solving the equation as a quadratic in p, we get

$$p = \frac{y \pm \sqrt{x^2 - y^2}}{x} \quad \text{i.e.} \quad \frac{dy}{dx} = \frac{y \pm \sqrt{x^2 - y^2}}{x}.$$

This is a homogeneous equation and hence we shall put $y = vx$.]

18. $\left(1 - y^2 + \frac{y^4}{x^2}\right)p^2 - 2\frac{y}{x}p + \frac{y^2}{x^2} = 0$ [P.U. 59H]

[**Hints**: The given equation reduces to

$$\left(p - \frac{y}{x}\right)^2 = p^2y^2\left(1 - \frac{y^2}{x^2}\right)$$

which $\Rightarrow$ $(px - y)^2 = p^2y^2(x^2 - y^2)$

$\Rightarrow$ $px - y = \pm py\sqrt{x^2 - y^2}$

$\Rightarrow$ $p(x \mp y\sqrt{x^2 - y^2}) = y$ which is homogeneous.]

ANSWERS

1. $(y - 3x - c)(y - 4x - c) = 0$
2. $(y - 3x - c)(y - 6x - c) = 0$
3. $(y - ax - c)(y - bx - c) = 0$
4. $(3y - x - c)(y - 3x - c) = 0$
5. $(y - e^x - c)(y + e^{-x} - c) = 0$
6. $(y + x^2 - c)(\log y - x - c) = 0$
7. $\left(\frac{1}{y} + x + c\right)(x^2 + y - c) = 0$
8. $(y - c)(2y - x^2 - c)(\log y - x - c) = 0$

9. $(xy - c)(x^2y - c) = 0$

10. $(2y - x^2 - c)(2x - y^2 - c) = 0$

11. $(y - x - c)(x^2 + y^2 - c^2) = 0$

12. $(y - cx)(x^2 - y^2 - c) = 0$

13. $(y^2 - x^2 - c)(xy - c) = 0$

14. $(y - cx^2)(y^2 + 3x^2 - c) = 0$

15. $(2y - x^2 - c)(y + x - 1 - ce^{-x}) = 0$

16. $\{2 \log y - (1 + \sqrt{5})\log x - c\}\{2 \log y - (1 - \sqrt{5})\log x - c\} = 0$

17. $\sin^{-1}\dfrac{y}{x} = \pm \log cx$

18. $\log\dfrac{x + \sqrt{x^2 - y^2}}{y} = c \pm y$